Problem 25.35

This is a more exciting version of 25.33. The voltage function is :

$$V = 5x - 3x^2y + 2yz^2$$

a.) What's E?

Using the del operator as before, we can write:

$$\vec{E} = -\vec{\nabla}V$$

$$\Rightarrow \quad = -\left[\left(\frac{\partial V}{\partial x}\right)\hat{i} + \left(\frac{\partial V}{\partial y}\right)\hat{j} + \left(\frac{\partial V}{\partial z}\right)\hat{k}\right]$$

$$\Rightarrow \quad = -\left[\left(\frac{\partial\left[(5x - 3x^2y + 2yz^2)\right]}{\partial x}\right)\hat{i} + \left(\frac{\partial\left[(5x - 3x^2y + 2yz^2)\right]}{\partial y}\right)\hat{j} + \left(\frac{\partial\left[(5x - 3x^2y + 2yz^2)\right]}{\partial z}\right)\hat{k}\right]$$

$$\Rightarrow \quad \vec{E} = -\left[\quad (5 - 6xy)\hat{i} \quad + \quad \left(-3x^2 + 2z^2\right)\hat{j} \quad + \quad (4zy)\hat{k} \quad\right]$$

$$\Rightarrow \quad \vec{E} = \left[(-5 + 6xy)\hat{i} + \left(3x^2 - 2z^2\right)\hat{j} - (4zy)\hat{k}\right] \text{ V/m}$$

1.)

b.) What is the magnitude of the electric field at (1, 0, -2) meters?

$$\vec{E} = \left[(-5 + 6xy)\hat{i} + \left(3x^2 - 2z^2\right)\hat{j} - (4zy)\hat{k}\right] \text{ V/m}$$

$$= \left[(-5 + 6(1)(0))\hat{i} + \left(3(1)^2 - 2(-2)^2\right)\hat{j} - (4(-2)(0))\hat{k}\right] \text{ V/m}$$

$$\Rightarrow \quad = \left[-5\hat{i} + -5\hat{j}\right] \text{ V/m}$$

Note that in polar notation, the electric field's magnitude is:

$$\left|\vec{E}\right| = \left[E_x^2 + E_y^2 + E_z^2\right]^{1/2} \text{ V/m}$$

$$= \left[(-5)^2 + (-5)^2 + 0^2\right]^{1/2} \text{ V/m}$$

$$\Rightarrow \quad \left|\vec{E}\right| = [7.07] \text{V/m} \ldots \text{ or } \left|\vec{E}\right| = [7.07] \text{N/C}$$

2.)